Residual life prediction under condition monitoring

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Abstract

Reliability assessment and remaining life prediction in the working processes of mechanical products, getting more attention of researchers, can reduce accidents and losses and help improve the preventive maintenance decision-making. This article presents two failure models, linear and exponential, to predict residual life distribution based on the degradation information of mechanical products. Parameters of the models can be estimated using maximum likelihood method. After the real-time monitoring information is acquired, residual life distribution should be updated constantly in order to improve accuracy of the prediction. Experiments were carried out on a double row cylindrical roller bearing to get the vibration information. It proved the validity of the aforementioned method and was applied to compare the two degradation models.

Keywords: reliability, residual life distribution, degradation model, double row cylindrical roller bearing

Introduction

In recent years, the reliability estimation study of mechanical parts or equipments have gained meaningful results at home and abroad. The feature information, which characterizes equipment's state, can be used to develop mathematical models. B E Cukor studied the combustion engines' wear rules of main parts under dynamic loading conditions and established mathematical models through regression of a large set of experimental data. F Y Wang et al carried out state detection on fans. Fitting process was conducted on the test data using least square method and the feasibility analyses of the limited operational life provided the expected effects [1]. A M Huang et al established a linear regression model of aviation hydraulic pump performance degradation with time using accelerated degradation test data and predicted the residual life [2]. The trend forecast based on Grey Model (GM) (1, 1) of grey system theory was introduced in [3, 4] and this article reported that the method was reliable. R Y Li et al predicted the failure rate of aircrafts of some airlines based on Auto Regressive Moving Average (ARMA) model and illustrated that ARMA model was suitable for forecasting failure rate [5]. Lu and Meeker adopted ARMA model to predict the failure time, and estimated the distribution of the failure time [6]. On the basis of this work, N Gebraeel et al developed an updated residual life distribution from the degradation signal in real time [7]. In general, the residual life prediction process based on the mathematical model can be shown as in Figure 1.

This paper establishes two models: the linear and the exponential. We assume that the distribution of the failure time is a Bernstein distribution. The distribution parameters are estimated using the maximum likelihood method and the mathematical models are gained to account for the remaining useful life (RUL) distribution. The residual life distribution can be updated by real-time data degradation continually.

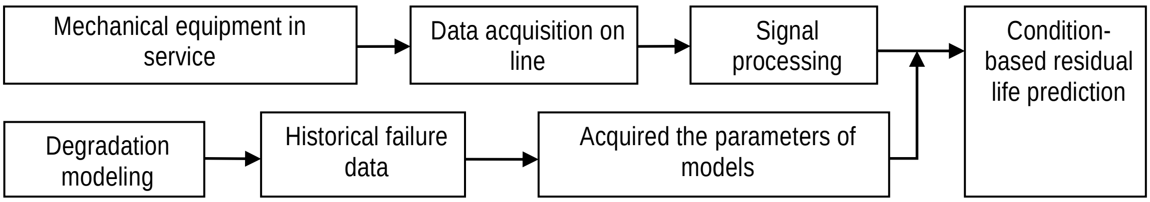

FIGURE 1 The process of residual life prediction

The models and the prediction algorithm

Establishing mathematical models by degradation signals, which can characterize the products' mechanical properties, to forecast RUL has been widely focused. In the paper, the level of degradation signals at times t_i is defined as $S(t_i)$, $i=1,2\ldots$, and the mathematical model

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can be expressed as $S(t_i) = f(t_i : \phi, \beta) + \varepsilon(t_i)$, where $\varepsilon(t_i)$ is the error term assumed to be i.i.d. normal random variables with mean 0, and variance σ^2. The deterministic parameter ϕ is a constant value in the whole degradation process. The parameter β is an assumed stochastic coefficient following a prior distribution. This paper describes two regression models and calculates the residual life distribution for each of them.

1.1 LINEAR DEGRADATION MODEL

The degradation model is given by the following equation

$$S(t_i) = \phi + \beta t_i + \varepsilon(t_i), \tag{1}$$

where the deterministic parameter $\phi = S(0)$. Assuming that $\pi(\beta)$ is the prior distribution of β, which is considered as i.i.d. $N(\mu_\beta, \sigma^2{}_\beta)$. We define $S_i = S(t_i)$, and assume that a unit degradation signal $S_1,\ldots,S_k$ has been observed up to a time level t_k. The conditional probability of the parameter β can be gained using Bayesian method and the updated distribution of β is

$$p(\beta / S_1,\ldots S_k) \propto p(S_1,\ldots S_k / \beta) \cdot \pi(\beta). \tag{2}$$

Because the error terms $\varepsilon(t_i)$ are i.i.d. $N(0,\sigma^2)$. So $(S(t_i) - \phi + \beta t_i)$ are following i.i.d. normal distribution with mean 0, and variance σ^2. The joint distribution of the observed signal can be expressed as:

$$p(S_1,\ldots,S_k / \beta) = \frac{1}{\prod_{i=1}^{k}\sqrt{2\pi\sigma^2}} \times \exp\left(-\sum_{i=1}^{k}\frac{(S_i - \beta t_i - \phi)^2}{2\sigma^2}\right). \tag{3}$$

Thus, the Eq. (3) is expressed as

$$p(\beta / S_1,\ldots S_k) \propto \exp\left(-\sum_{i=1}^{k}\frac{(S_i - \beta t_i - \phi)^2}{2\sigma^2}\right) \cdot \exp\left(-\frac{(\beta - \mu_\beta)^2}{2\sigma^2{}_\beta}\right) \propto \frac{1}{\sqrt{2\pi\sigma^2{}_\beta}} \exp\left(-\frac{(\beta - \mu_\beta)}{2\sigma^2{}_\beta}\right).$$

The parameters of the updated distribution of β are given by:

$$\mu_\beta = \frac{\sigma^2{}_\beta \sum_{i=1}^{k}(S_i - \phi)t_i + \sigma^2 \mu_\beta}{\sigma^2{}_\beta \sum_{i=1}^{k} t_i^2 + \sigma^2}, \text{ and } \sigma^2{}_\beta = \frac{\sigma^2 \sigma^2{}_\beta}{\sigma^2{}_\beta \sum_{i=1}^{k} t_i^2 + \sigma^2}. \tag{4}$$

At some time level t in the future, the level of the degradation signal can be expressed by the random variables $S(t + t_k)$. The mean and the variance can be given by:

$$\mu(t + t_k) = \phi + \mu_\beta t; \ \sigma^2(t + t_k) = \sigma^2{}_\beta t^2 + \sigma^2. \tag{5}$$

We define the predetermined failure threshold as D and the distribution of the residual life T_R can be determined from the conditional probability as follows:

$$F(T_R \le t \mid S_1,\ldots,S_K) = P(S(t + t_k) \ge D \mid S_1,\ldots,S_k) = \Phi\left(\frac{\mu(t + t_k) - D}{\sigma(t + t_k)}\right), \tag{6}$$

where $\Phi(\bullet)$ is the CDF of a standardized normal random variable. Factually, the value of a unit's residual life can never be negative, so the negative values of the remaining life should be precluded. Figure 2 is the probability density function of the residual life and the area of the shaded part should be eliminated. Thus, the CDF of the residual life is:

$$p(T_R \le t \mid S_1,\ldots,S_K) = \frac{\Phi(\frac{\mu(t+t_k)-D}{\sigma(t+t_k)}) - \Phi(\frac{\mu(t_k)-D}{\sigma(t_k)})}{1-\Phi(\frac{\mu(t_k)-D}{\sigma(t_k)})} . \quad (7)$$

2.2 EXPONENTIAL DEGRADATION MODEL

The exponential degradation model is widely used when the cumulative damage is significant [7, 8]. The form of the exponential model is defined as:

$$S(t_i) = \phi \exp(\beta t_i + \varepsilon(t_i) - \frac{\sigma^2}{2}) , \quad (8)$$

where the parameter ϕ is a constant, and we assume that $\varepsilon(t_i)$ are i.i.d. normal random variables with mean 0, and variance σ^2 and the parameter β is a stochastic coefficient following a prior distribution, which is i.i.d. $N(\mu_\beta, \sigma^2{}_\beta)$, as in linear degradation model.

Now, we can transform the exponential model into a linear form as follows:

$$L_i = \ln S(t_i) = \phi' + \beta t_i + \varepsilon(t_i) , \quad (9)$$

where $\phi' = ln\phi - \frac{\sigma^2}{2}$, and ϕ' is also a constant.

To gain the parameters of the updated distribution of β is analogous to that of the linear modelling method introduced above and is a normal distribution. Similarly, the mean and the variance of the random variable $L(t+t_k)$ are

$$\mu(t+t_k) = \phi' + \mu_\beta t , \text{ and } \sigma^2(t+t_k) = \sigma^2{}_\beta t^2 + \sigma^2 . \quad (10)$$

3 The estimation of the prior distribution of the stochastic parameters

The prior distribution parameters should be estimated first when the residual life distribution of the mechanical products is updated. N. Gebraeel et al. demonstrate that there are almost no impacts on forecast accuracy of the models' parameters using aforementioned monitoring information and historical failure data [8]. This article introduces the computing method of the prior distribution parameters using historical failure data only. Degradation model can be expressed as:

$$W(t) = \lambda + \omega t , \quad (11)$$

where λ and ω are random variables and $W(t)$ is the level of degradation signals. We assume as $\lambda \sim N(\mu_\lambda, \sigma_\lambda)$ and $\omega \sim N(\mu_\omega, \sigma_\omega)$, thus, $W(t) \sim N(\mu_\lambda + \mu_\omega t, \sigma^2{}_\lambda + \sigma^2{}_\beta t)$.

The failure time distribution is expressed as:

$$F(t) = p(T_L \le t) = P(W(t) \ge D) = 1 - \Phi(\frac{D-(\mu_\lambda + \mu_\omega t)}{\sqrt{\sigma^2{}_\lambda + \sigma^2{}_\omega t}}) , \quad (12)$$

where T_L is the unit's life, which is defined as a random variable. The unit is regarded to failure when $W(t)$ reaches the threshold Bernstein distribution is widely used in the residual life prediction. The failure time distribution in Eq. 14 can be expressed as a 2-parameter Bernstein distribution [9]:

$$f(t) = \frac{c}{\sqrt{2\pi a t^2}} \exp\{-\frac{1}{2a}(1-\frac{c}{t})^2\} . \quad (13)$$

Now, estimate the stochastic parameters by maximum likelihood estimation method. The likelihood function is:

$$L = \prod_{i=1}^{n} f(t_i) = \frac{c^n}{(2\pi a)^{n/2}\prod_{i=1}^{n} t_i^2} \exp(-\frac{1}{2a}\sum_{i=1}^{n}(1-\frac{c}{t_i})^2) . \quad (14)$$

After applying logarithm, it can be given by:

$$\ln L = n \ln c - \frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln a - 2\sum_{i=1}^{n} t_i - \frac{1}{2a}\sum_{i=1}^{n}(1-\frac{c}{t_i})^2 , \quad (15)$$

where $\{t_i\}$ is the set of failure times data.

The results are:

$$\hat{c} = \frac{n}{\sum_{i=1}^{n} 1/t_i}, \text{ and } a = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{\log t_i} - \frac{1}{\hat{c}}\right)^2 . \quad (16)$$

The parameters c and a are used to evaluate the mean μ_ω and variance σ^2_ω of the parameter ω, and

$$\mu_\omega = \frac{D-\lambda}{c}, \text{ and } \sigma^2_\omega = \mu^2_\omega a . \quad (17)$$

The estimated parameter ω is replaced by β, which is a random parameter of the established models (linear and exponential). Thus, the prior distribution of the degradation models is determined. The parameters of the updated distribution can be obtained according to the actual degradation process of the units and the residual life distributions based on the given conditions are obtained.

4 An example

The vibration data are taken from the Rexnord ZA-2115 double row cylindrical roller bearings, using NI DAQCard-6062E data acquisition card, PCB 353B33 acceleration sensor, and the data acquisition software developed by LabVIEW. The sampling frequency is 20 kHz and the sampling length is 20480. It is required to calculate the vibration intensity through each of the 10min sampling data while the sampling interval is kept 10min. In order to verify the performance of the model, the model is established and analysed through one of the 180 vibration intensity data, which are shown in Figure2.The vibration data are taken from the Rexnord ZA-2115 double row cylindrical roller bearings, using NI DAQCard-6062E data acquisition card, PCB 353B33 acceleration sensor, and the data acquisition software developed by LabVIEW. The sampling frequency is 20 kHz and the sampling length is 20480. It is required to calculate the vibration intensity through each of the 10min sampling data while the sampling interval is kept 10min. In order to verify the performance of the model, the model is established and analysed through one of the 180 vibration intensity data, which are shown in Figure 2.

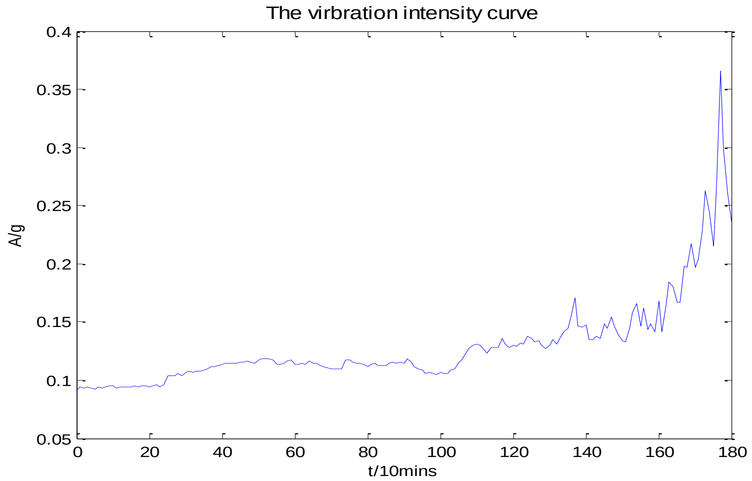

FIGURE 2 The vibration intensity curve

According to the modelling method and the calculation method described above, the prior distribution parameters of the two degradation models (linear and exponential) are assessed using historical data of failure, which are shown in the Table 1.

TABLE 1 The estimated parameters of the prior distribution

	The value of ϕ (ϕ')	Distribution parameter of β	
		μ_β	σ^2_β
Linear Degradation model	0.0916	5.8661×10^{-4}	1.0978×10^{-6}
Exponential Degradation model	-2.4206	0.0044	1.9529×10^{-8}

The experimental results of the rolling bearing vibration intensity data are used to update the residual life distribution. Figure 3 illustrates evolution of the updated residual life distributions. Meanwhile, the prediction errors of the linear model and the exponential model are compared and the results are shown in Figure 4. In the prediction, the median of the distribution of the residual life is regarded as estimated remaining life [8].

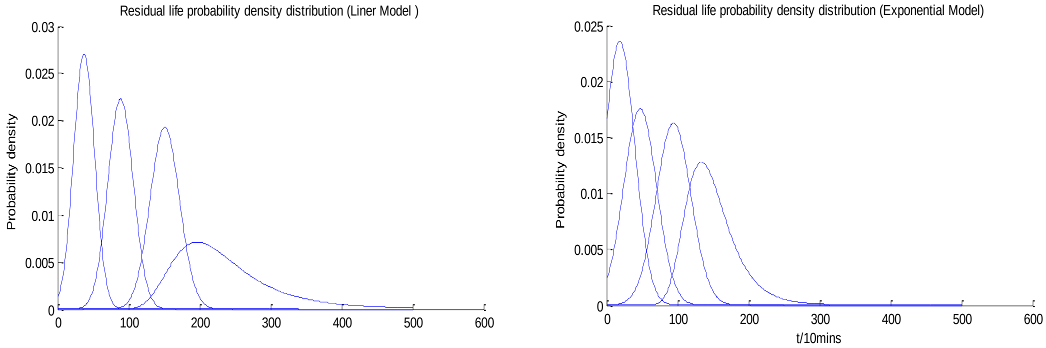

FIGURE 3 Evolution of residual life distributions

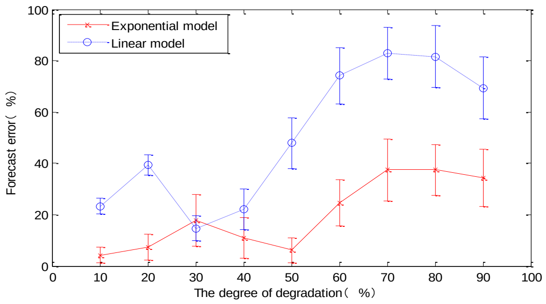

FIGURE 4 The errors of linear model and exponential model

From the experiment results, three observations are noteworthy:

1) The probability density distributions of the residual life are more and more concentrated with times. In the beginning, the probability density distributions are dispersive in both linear model and exponential models.
2) The prediction errors of the linear model are significantly larger than those of the exponential model, which we can be quantified from Figure 5. Because the rolling bearings failures are mostly caused by fatigue and the fatigue is a cumulative process. Thus, the exponential model is more suitable for the degradation process of the rolling bearings.
3) With increasing degree of degradation, the accuracy of the forecast seriously declines. When the bearing is close to failure, the intensity of vibrations is sharply increased, which is observable in Figure 2. This implies that the exponential model is not suitable anymore for this stage.

5 Conclusions

This paper discusses a residual life prediction method of a unit or a system through adding online performance measurements continually. Two degradation models (linear and exponential) are established in the paper and the parameters of the two models are estimated by means of a widely used Bernstein distribution. The distributions of the residual life are updated with change of the units' state. The vibration intensity information extracted from the operation of double row cylindrical roller bearings validates the feasibility the method. Moreover, from comparison of the prediction errors, it can be concluded that the exponential model is more suitable than the linear model. Research on reliability of mechanical components or equipment can predict the future performance, which is helpful for preventive maintenance decision-making and improving quality of the system security assessment.

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